Trying to measure some microscopic quantities through macroscopic measurements.

What is microscopic and what is macroscopic?

Micsroscopic means small scale, while macroscopic means normal scale.

We are macroscopic. An atom is microscopic.

Of course there isn't an exact size that sizes above which are macroscopic and those below which are microscopic.

But there are rough sizes. Roughly speaking, those which we can see by naked eye or simple magnifiers are called macroscopic. Those which we can't see by these means (as they are too small) are called microscopic.

A typical size could be about a micron (one micrometer): above this macroscopic, below this microscopic.

micro means 10^(-6)

Consider two quantities:

The typical distance between two neighboring atoms in a liquid or solid, and
the latent heat of evaporation

atoms in a solid or liquid

$l$: the typical distance between two neighboring atoms

The latent heat?
When a liquid is heated, its temperature rises.

At some point, the liquid starts boiling. At this point, the temperature remains fixed.
But the liquid is still absorbing heat.

What happens to that heat? It makes the liquid change into gas (vapor).

The heat which is needed to change the liquid into gas, is called the latent heat
(of evaporization).

A similar thing happens in melting (a solid changing into a liquid):
The heat needed to change the solid into liquid is called the latent heat of melting.

Why latent? Because the temperature isn't changing. So the heat is "hidden".

$\longrightarrow \ell$

What is the relationship between L (the latent heat) and l (the distance between
neighboring atoms in a liquid or solid?) ?

By the way, l is a microscopic quantity, and L is a macroscopic quantity.

One cannot see the atoms or the distance between them, with naked eye or simple magnifiers.

But what can measure the latent heat with normal-size instruments, without the need
to study individual atoms.

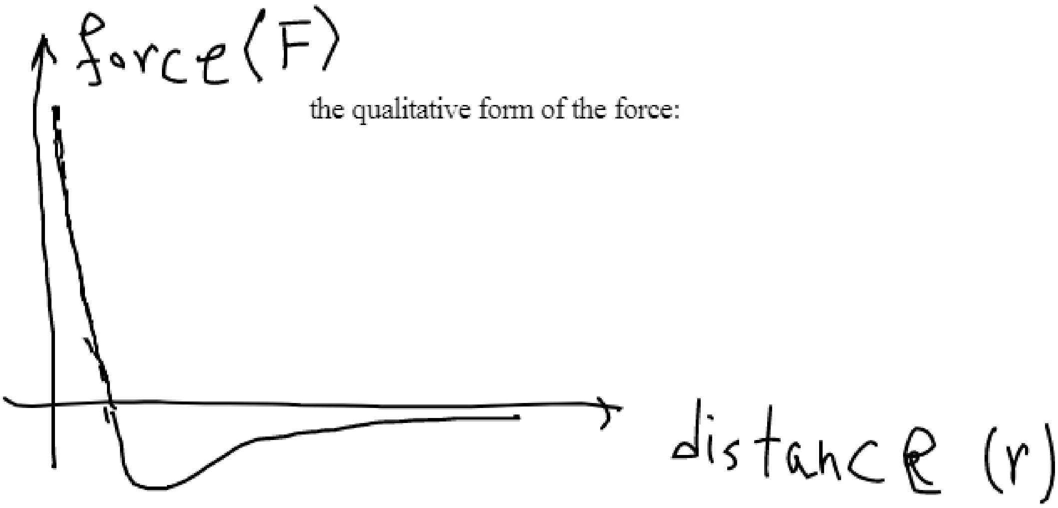
The aim is to relate these two quantities, and use their relationship to calculate
(or estimate) the microscopic one through a measurement of the macroscopic one.

Why is there a latent heat? Why is heat (energy) needed to change a liquid into a solid?

Answer: when two atoms (or molecules) are near each other (their distance is about the size of atoms or molecules) there is an interaction between them.

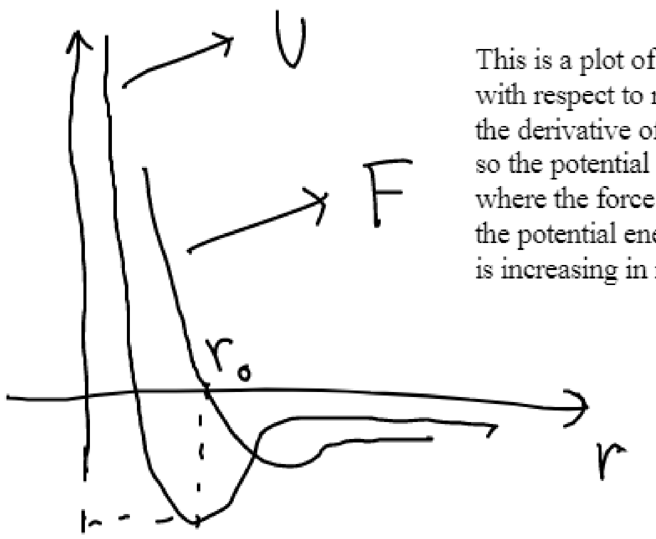
There is a force between them. This force is attractive, when the distance is larger than some value, and repulsive when the distance is smaller than that value:

the qualitative form of the force:




This force is related to a potential energy U

$$F(r) = -\frac{dV}{dr}$$




This is a plot of both the force and the potential energy with respect to r: Where the force is positive, the derivative of the potential energy is negative, so the potential energy is decreasing in r; where the force is negative, the derivative of the potential enegy is positive, so the potential energy is increasing in r.

$$U(r_0) = -V_0$$

At $r_0$, the force is zero and the potential energy has a minimum. The value of the potential energy at this point is denoted by $U_0$ $(-U_0)$

In a liquid, each atoms has (on the average) g neighbors. g can be different for different substances, but it is a number which is not very large, say 4,5, 6, ...

Corresponding to each pair of neighboring particles, there is a potential energy $(-V_0)$

So the total potential energy is $(-V_0) \times$ (the number of neighboring pairs.)

The number of neighboring pairs:
For each particle, there are g neighbors. So for N particles there are (g N) pairs.

But in this calculation, each pair has been counted two times.

1 2

Particle 2 is a neighbor of particle 1, so the pair (1 2) has been counted when neighbors of 1 have been studied.
But particle 1 is a neighbor of particle 2, so the ~~smae~~ same pair (1 2) has also been counted when neighbors of 2 have been studied.

So the total number of neighboring pairs is $\frac{gN}{2}$

And the total potential energy is (- g N U_0 / 2)

Now compare N atomes in the liquid states and N atoms in the gas state.

For the liquid state, there is a potential energy: $(- g N U_0 / 2)$

For the gas state, there is no potential energy. Or one can say, the potential energy is 0.

So (the energy of gas) - (the energy of liquid) = $0 - (- g N U_0 / 2) = g N U_0 / 2$

All of these quantities are positive. If the liquid is to be turned into the gas, some energy is to be provided.

And this energy is proportional to the number of particles:

$L = g N U_0 /2$

This (L) is the latent heat (of evaporation).

While g and 2 are dimensionless numbers not far from 1,
$U_0$ has the dimension of energy, and is difficult to be measured directly.

Because $U_0$ is the interaction energy between two (microscopic) partilces,
say two molecules.

But L itself is a macroscopic quantity.

One can measure the latent heat of evaporation for a macroscopic bulk of a liquid, for example one kilogram of water.

While N (the number of particles) and U_0 (the interaction energy between two particles) are microscopic, their product (N U_0) is a macroscopic quantity.

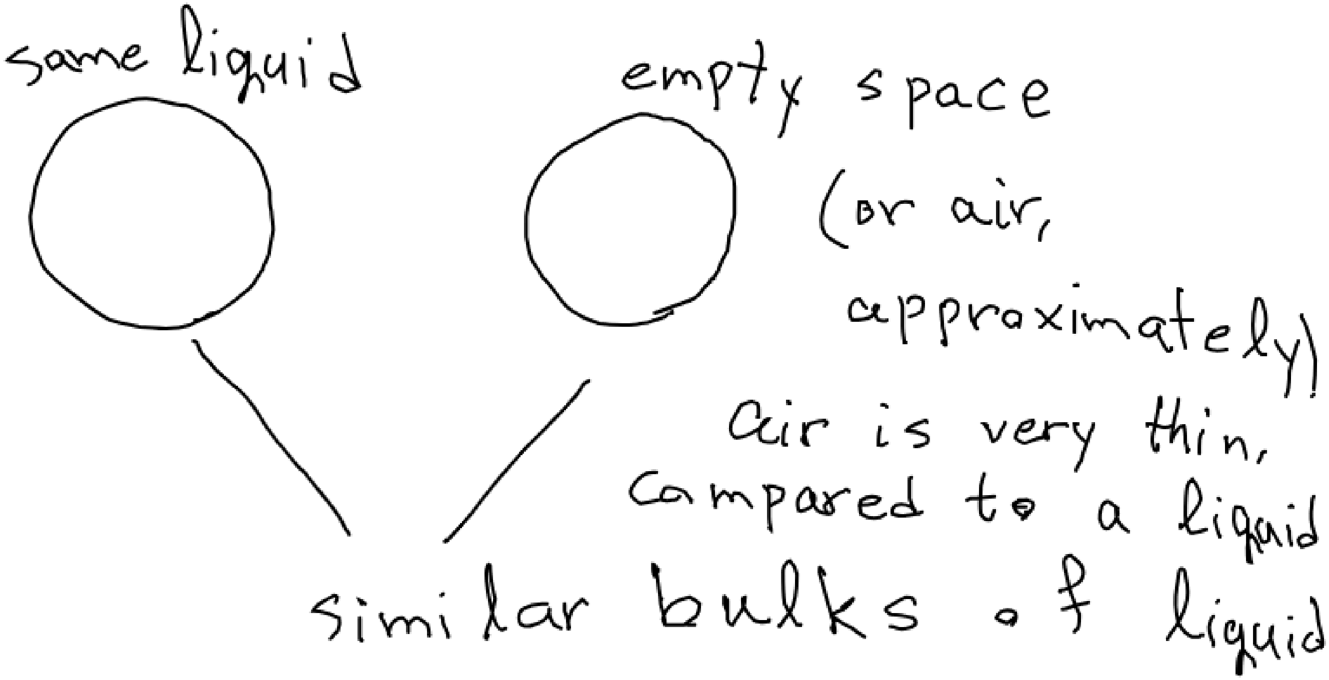
So, measuring the latent heat (macroscopically) provides information about two microscopic quantities (N and U_0), but doesn't give the values of both: it only gives the product (N U_0).

Another measurement is needed to find both N and U_0.

Another macroscopic quantity, which is called the surface tension:

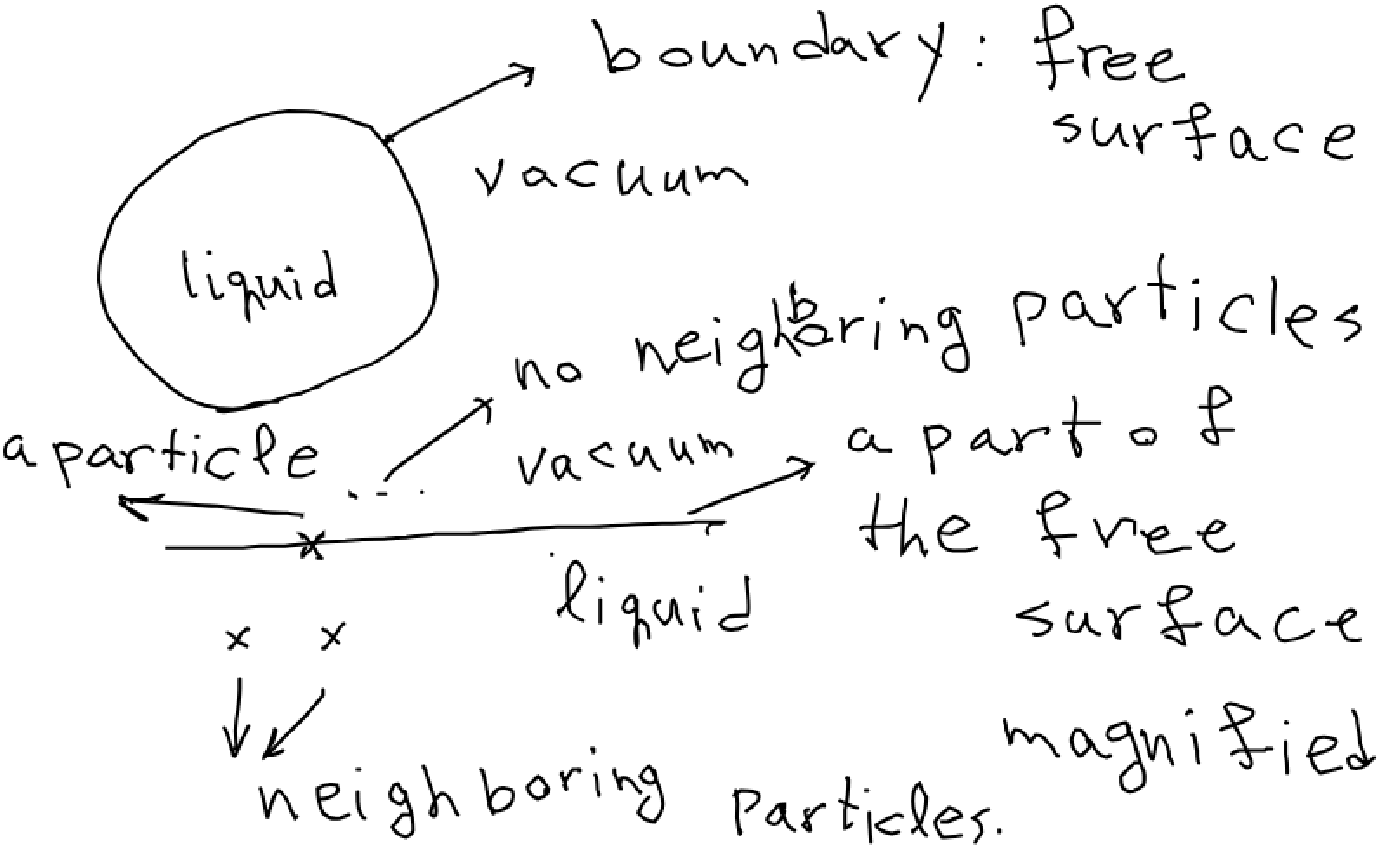
So what is the surface tension?

Consider two similar bulks of liquid, one inside the same liquid, another inside empty space (vacuum)




Air contains much fewer particles per volume, compared to a liquid, so one can consider air to be empty (approximately).

The difference between these two bulks (of liquid):
For the one sorrounded by vacuum, the particles on the boundary have fewer neighbors:



So the number of neighbors for a particle on the bounday (the free surface) is smaller than the number of neighbors for a particle in the bulk. The number of neighboring particles on the boundary is g'.

$g' < g$.

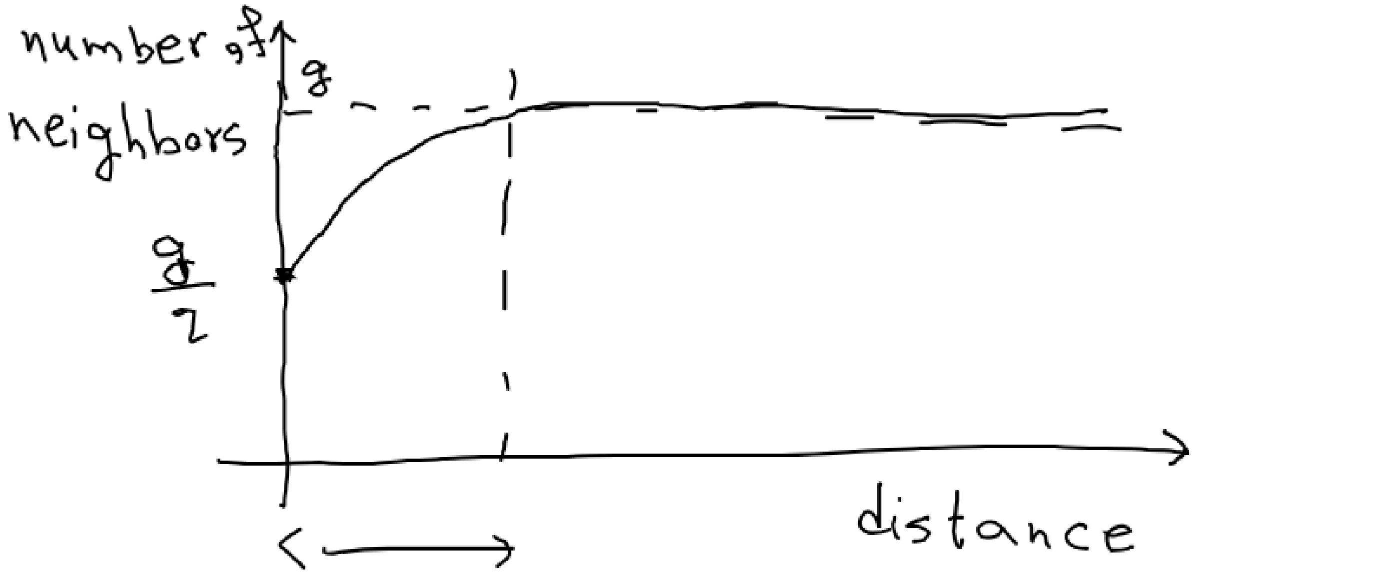
In fact, exactly on the free surface,

$g' = (g/2)$

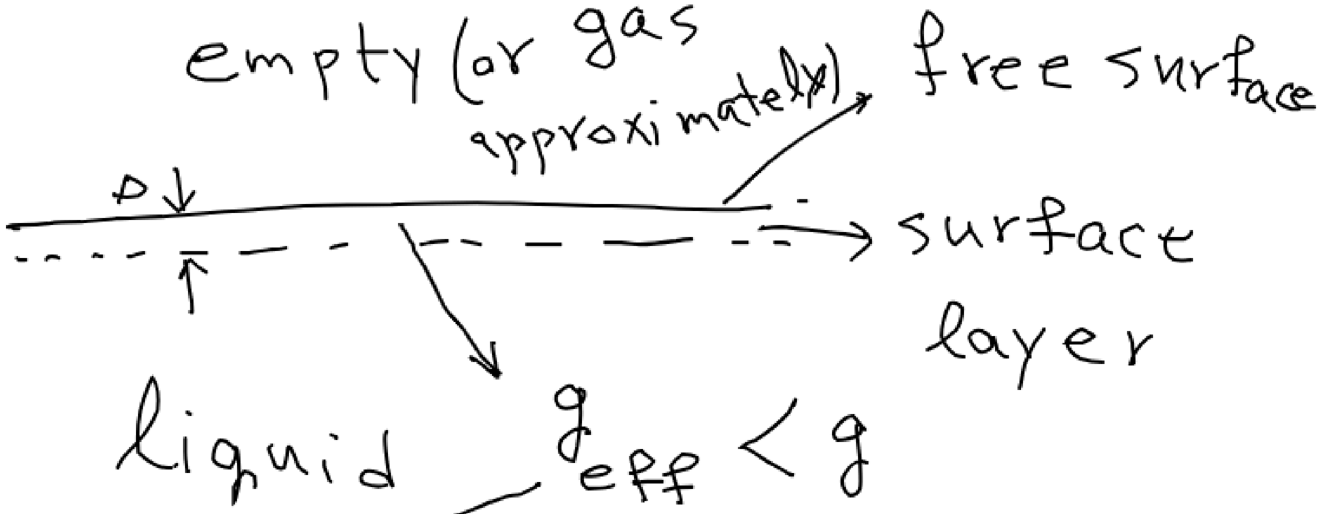
Becaues half the would-be neighbors should have been on the vacuum side, and these are lost.

As one moves from the free surface towards the bulk, the number of neighbors increases, so that eventually it reaches the bulk value g.

One can draw a picture for this:




The plot corresponds to a layer of width $\Delta$, in which the number of neighbors is less than g




This effective $g$ is the averge of the number of neighbors in the surface layer

For a liquid inside the same liquid, there is no such surface layer.

So the total energy of a liquid with a free surface, is different from that of a same bulk of liquid without a free surface.

(The energy with the free surface) - (The energy without the free surface) =

$$(-g_{eff} \mathcal{N} U_0) - (-g \mathcal{N} U_0) = \overbrace{\left(\frac{g - g_{eff}}{2}\right)}^{>0} \overset{>0}{\mathcal{N}} \overset{>0}{U_0}$$

$\mathcal{N}$ = The number of particles in the surface laye

All three factors are positive, so this energy difference is positive. It is called the surface energy.

What is the value of $\mathcal{N}$?

$$\mathcal{N} = \frac{N}{V} \times \text{(the volume of the surface layer)}$$

the volume of the surface layer = (the thickness of the surface layer) times (the surface area of the free surface)

$$= \Delta \times S$$

$$\text{the surface energy} = \left[\left(\frac{g - g_{eff}}{2}\right)\frac{N}{V}U_0 \Delta\right] S$$

The quantity inside the brackets is called the surface tension:

$$\tau = \frac{g - g_{eff}}{2}\,\frac{N}{V}U_0\,\Delta$$

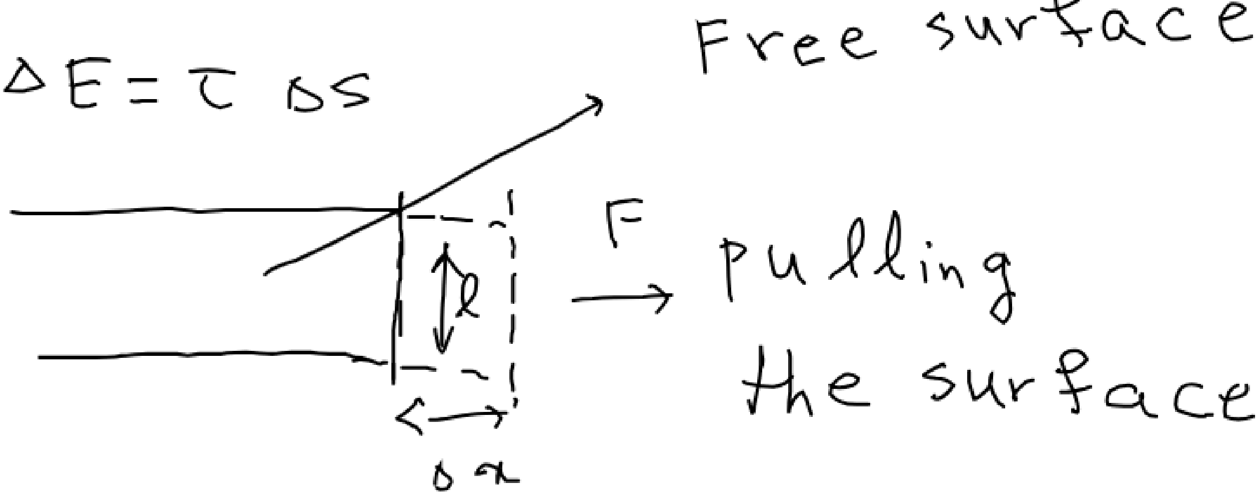
$$E = \tau S$$

the surface energy

How to measure the surface tension?

Increasing the surface area requires energy:

$$\Delta E = \tau \Delta S$$




$l$: width of the free surface

$F$: the pulling force

This $F$ does work

This work is the same as the energy increase:

$$W = F \Delta x$$

$W$: Work

$\Delta x$: displacement

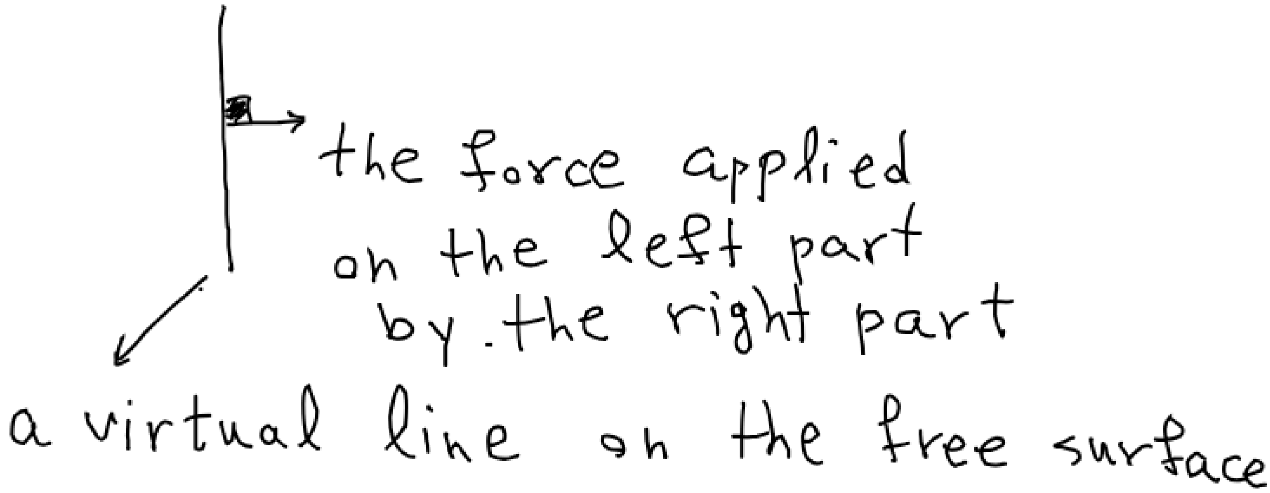
$$W = \Delta E = \tau \Delta S$$

$$\Delta S = l \Delta x$$

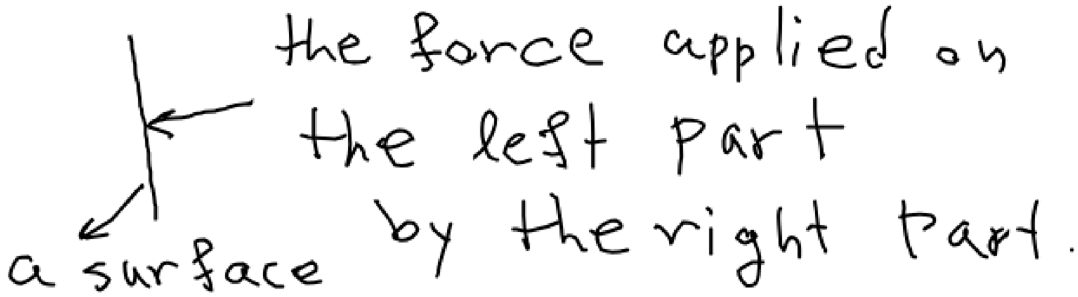
$$\tau l \Delta x = F \Delta x$$

$$\tau l = F \qquad \tau = \frac{F}{l}$$

The surface tension is the (pulling) force per length, and it is normal the line.




Comparing with the pressure:
The pressure is the (pushing) force per surface area, and it is normal to the surface.



$$L = \frac{g}{2} N U_0$$ the latent heat of evaporation

$$\tau = \frac{g - g_{eff}}{2} \frac{N}{V} U_0 \Delta$$ the surface tensino

$$\frac{L}{V} = \frac{g}{2} \frac{N}{V} U_0$$ the latent heat of evaporation per volume

$N, U_0, \Delta$ : micrascopic
$V$ : macrascopic

$g, g_{eff}, 2$ are of the order 1

Dividing $\tau$ by $(L/V)$, the factor

$\left(\frac{N}{V} U_0\right)$ is cancelled:

$$\frac{\tau}{\left(\frac{L}{V}\right)} = \frac{g - g_{eff}}{g} \Delta$$

$\Delta$ is the width of the surface layer. It is a microscopic quantity. It is of the order of the size of a particle

But $\tau$ and $\frac{L}{V}$ are macroscopic.

So the size of a particle (an atom, a molecule) is of the order of $\frac{\tau}{\left(\frac{L}{V}\right)}$

Numerical values.

The latent heat of water = 2300 J / g

The volume of 1 g of water is 1 cubic centimeter, which is $10^{-6}\ m^3$

$$\frac{L}{V} = \frac{2300\ J}{g} \times \frac{1\ g}{1\ (cm)^3} = \frac{2300\ J}{(1\ cm)^3}\ \frac{1\ (cm)^3}{10^{-6}\ m^3}$$

$$\frac{L}{V} = 2.3 \times 10^{9}\ \frac{J}{m^3} \sim 2 \times 10^{9}\ \frac{J}{m^3}$$

$$\sim 10^{9}\ \frac{J}{m^3}$$

The surface tension of water is about 7 centiNewton per meter.

$$\tau = 7\ \frac{cN}{m} = 0.07\ \frac{N}{m} \sim 0.1\ \frac{N}{m}$$

I am performing a very approximate calculation,only orders of magnitude are kept.
The final result is

$$\Delta \sim \frac{0.1}{10^9} \frac{\frac{N}{m}}{\frac{J}{m^3}} = 10^{-10} \; m^2 \frac{N}{J}$$

$$J = N\,m \qquad \Delta \sim 10^{-10} \; m$$

The final result (while only an order of magnitude) is remarkably good.
The size of a particle (an atom or a molecule) is about 10^(-10) meter,
which is one Angstrom.

This is the correct (order-of-magnitude) result.

And this correct miscroscopic result was obtained through macroscopic measurements.

The measurements of the surface tension and the latent heat don't require "seeing"
atoms or molecules.