In the last session, we talked about scaling relations. Roughly speaking, scaling relations are those relations which don't need scales.

The form of a scaling relation between X and Y, turned out to be

$$Y = b X^c$$

Where b and c are constants.

Examples were the relations between the volume and the surface area with the linear size:

The volume is proportional to the linear size to the power 3.

The surface area is proportional to the linear size to the power 2.

Going further, the mass is proportional to the linear size to the power 3 (assuming that the density is fixed).

$$m = \rho V$$

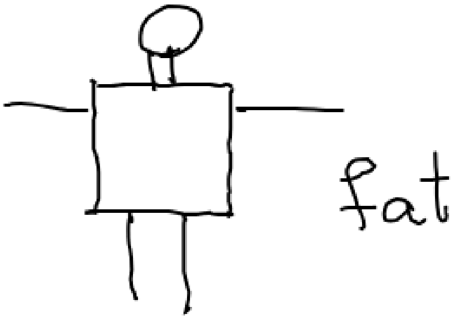
m is the mass, rho is the density, and V is the volume.

$V = b\, l^3$ $\quad$ $m = (\rho\, b)\, l^3$ $\quad$ l is the linear size.

Using these, let's study a quantity called BMI.

BMI: Body Mass Index

This is a number which tells whether a person is thin or fat.



The definition of BMI is the following.

BMI = [mass / (k g)] / (height / m)^2

Example: mass = 60 k g height = 1.7 m

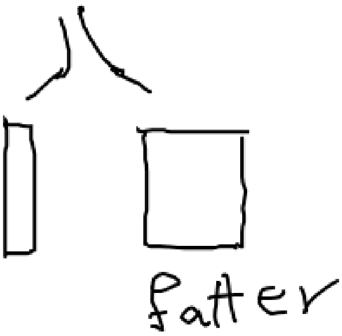
BMI = 21 (20.76) rounded to 21

Why this particular combination?

Some observations:

Two persons with the same height: The one with a bigger mass is fatter:

Two persons, neglecting their arms, legs and heads



Two persons with the same mass: The one with a bigger height is thinner:

These observations show that BMI should be increasing with mass and decreasing with height:

BMI should increase when the mass increases, and decrease when the height increases.

So, assuming a power law for the relation between BMI and the mass and ~~teh~~ the height, BMI should be

$$\mathrm{BMI} = c\,(\mathrm{mass})^a/(\mathrm{height})^b$$

where c and a and b are positive constants.

The larger the BMI, the fatter the person.

If an increasing function of BMI is used instead of BMI, the above doesn't change.

For example, multiplying BMI by a positive constant, or raising it to a positive exponent, doesn't change that relation.

$\mathrm{BMI} \to \alpha\,\mathrm{BMI}$ $\qquad \alpha > 0$

$\mathrm{BMI} \to (\mathrm{BMI})^\beta$ $\qquad \beta > 0$

These show that the actual value of c is not important.

Also, between a and b, only the ratio (b/a) is important, because one can use $(\mathrm{BMI})^{(1/a)}$ instead of (BMI).

$\mathrm{BMI}' = (\mathrm{BMI})^{(1/a)}$ $\qquad \mathrm{BMI}' = c'\,\mathrm{mass}/(\mathrm{height})^{(b/a)}$ $\qquad c' = c^{(1/a)}$

The only important thing is (b/a).

So,one can write the following for BMI.

BMI = c' mass /height^(b') = [c' k g m^(-b')] [mass / (k g)] / (height / m)^(b')

The relation has been multiplied and divided by (k g) and m^(-b').

It was seen that the overall multiplicative constant, here [c' kg m ^(-b')], is not important.

One can set it equalto one. The result is

BMI = [mass / (k g)] / (height / m)^(b')

The only important thing is the exponent b', which is a positive constant.

The only question: What is the value of b'?

Or, why is b' = 2 (in the usual definition of BMI)?

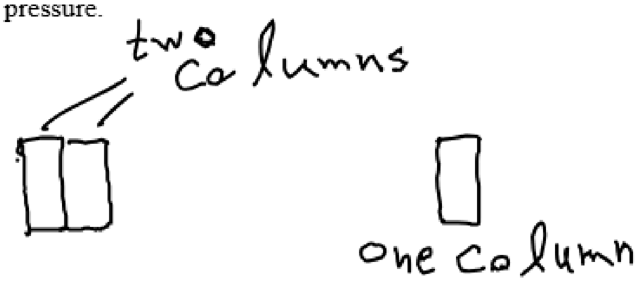
To see the reason (for the exponent being equal to 2),

let's ask what is the important physical quantity in being fat or thin.
What happens to the body when someone is fat?

When someone is fat, the organs of her or his body are under pressure.

It is the pressure which is inportant, not the weight itself.

Example: consider two similar columns beside each other.



And compare this with one of those two columns.

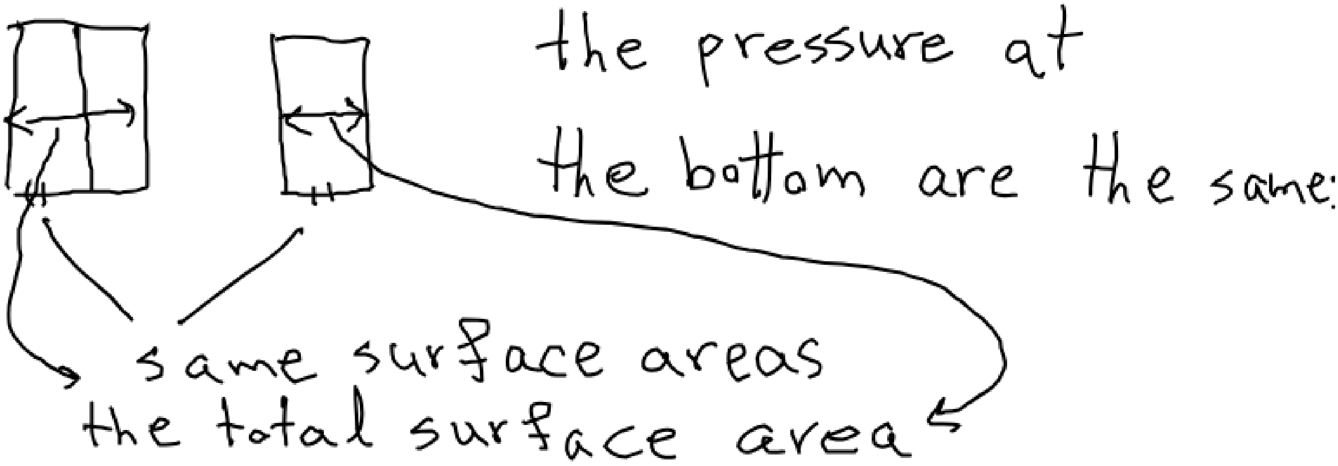
The weight of the set of two columns is twice (two times) the weight of one column.

But what about the probability of these falling apart under ~~thei~~ their weight?

It is not true that the falling-apart probability of the set of two columns is bigger
that that of the single column.

The probabilities are the same. The reason is that the force acted a small piece at the bottom
is equal to the pressure times the surface area of that piece.

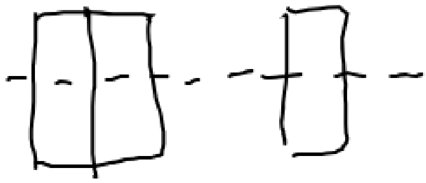
So for two similar small pieces, one is the set of two columns, the other in the one column, the forces are the same, as the surfaces areas are teh same, and the pressures are the same:



The pressure = the weight / (the total surface area)

The total surface area is doubled for the two columns, and the weight is doubled for the two columns as well.

This is true (that the pressures are the same), not only at the bottom, but at any height (same height for the two columns and the one column).



The pressures are the same at the broken line.

Now we know that the important quantity is the pressure.

The pressure is equal to the weight divided by the surface are.

The weight is equal to the gravity acceleration times the mass.

The surface area is proportional to a length size ~~two~~ to the power 2

P = w / S $\quad$ w = g m $\quad$ S = k h^2

P is the pressure, w is the weight, g is the gravity acceleration, m is the mass,
S is the surface area (of a cross section of the body), h is the height (a length size),
and k is a constant.

a cross section of the body

So: $P = \frac{g}{k} \frac{m}{h^2}$

This is a constant

$P \propto \frac{m}{h^2}$

$$P \propto BMI$$

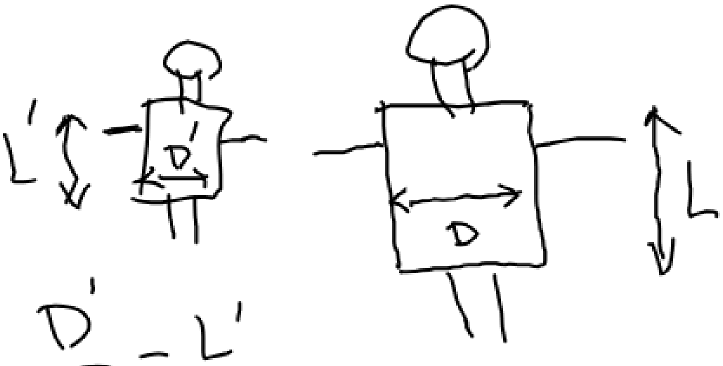
This is the reason for the particular definition of BMI:
The pressure is the important factor, and the pressure is proportional to the mass divided by (the square of the height).

One result of this, is that if two persons are similar to each other (the ratio of their length sizes are the same for any length size), then the bigger person has a bugger BMI. So the bigger person is fatter.

i

similar triangles

$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$$



$$\frac{D'}{D} = \frac{L'}{L} = \cdots$$

similar persons: the right person is fatter the right body is under more pressure.

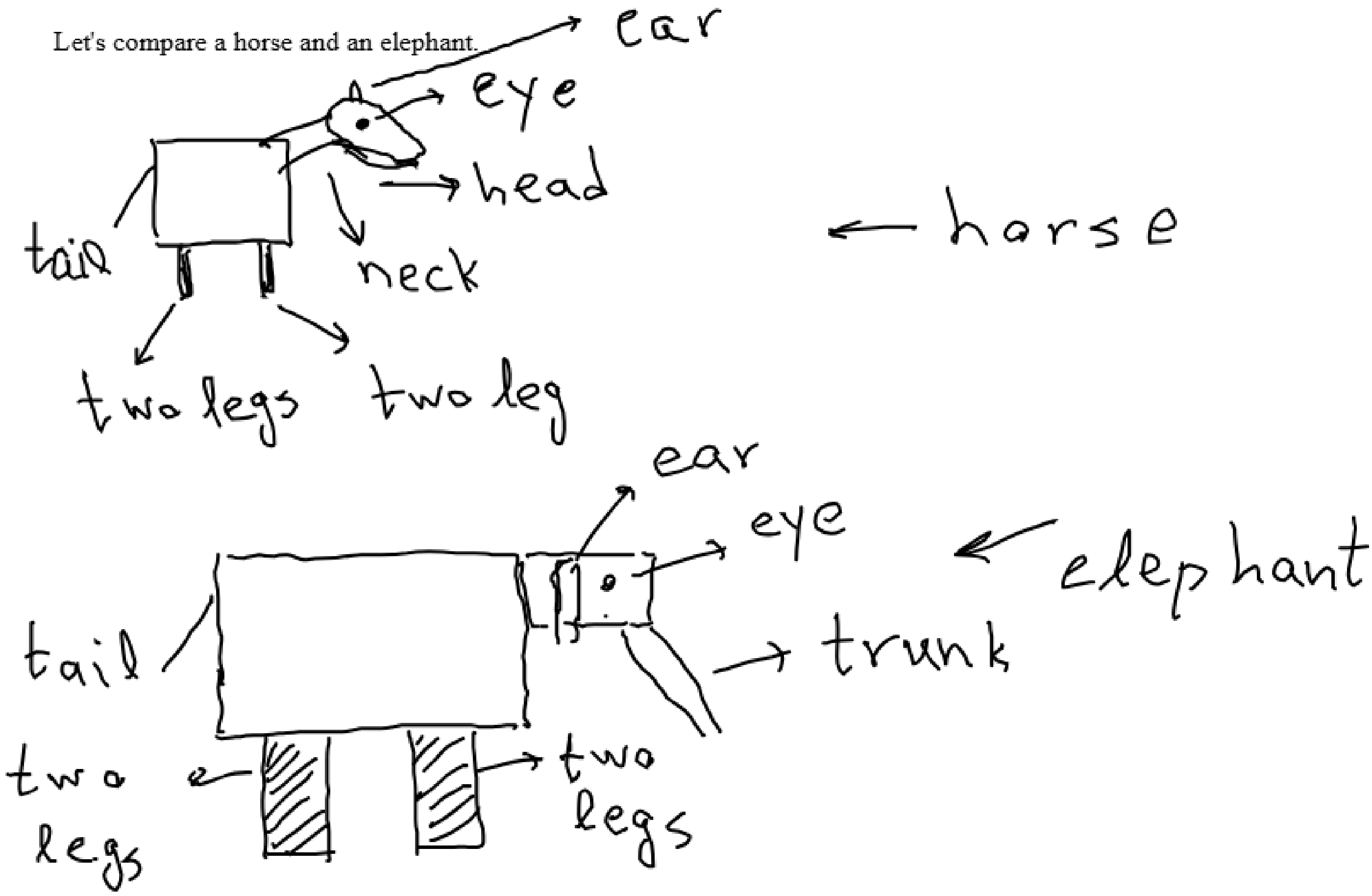
Why?

Because for similar persons, the mass is proportional to h^3.
So BMI is proportional to (h^3 / h^2).
That is, BMI is proportional to h.

The bigger the person, the bigger h, the bigger BMI.

In fact a similar thing happens for the animals.

Let's compare a horse and an elephant.



The elephant is much bigger than a horse.

The mass of a horse can be up to several hundred k g.
The mass of an elephant can be up to several thousand k g.

This means a mass ratio of about 10, or a length size ratio of more than 2.

If the elephant and the horse were similar, the elephant would be in trouble,
with a BMI more than two times the BMI of the horse.

looks

But if one ~~llooks~~ at the elephant and the horse more closely,
it is seen that the legs of the elephant are much thicker (proportionally)
~~that~~ the legs of the horss.

than

That is, if the height of the elephant is two times the height of the horse,
the thickness of the leg of the elephant is much more than the thickness of the leg og the horse.

two times