one of the causes of the force pn a body moving inside a fluid, is the collision of the particles of the fluid with the body.

When a particle collides with the body, it transfers some momentum to the body.
The linear momentum is the product of the mass and the velocity:

$$\underbrace{\vec{P}}_{\text{linear momentum}} = \underbrace{m}_{\text{mass}}\vec{V} \longrightarrow \text{velocity}$$

$$\underbrace{\vec{F}}_{\text{force}} = \frac{\Delta\vec{P}}{\Delta t} \longrightarrow \text{time}$$

The number of the particles

The mass of a particle

The speed of a particle

The size of the body

Instead of the (total) number of the particles, the number density is used.

The number density, is the number of the particles per volume.

The number density (of the fluid particles)

The mass of each fluid particle

The speed of each fluid particle

The size of the body

Two particles each of mass m and speed v, trasnfer the same amount of momentum as a single particle of mass (2 m) and speed v.

Instead of the number density, and the mass, their product is used.

There are N particle in a volume V. So the number density is (N/V).

The mass of each particle is m. So the total fluid mass inside the volume V is (m N).

And the mass density is (m N/V), which is equal to [m (N/V)], that is the mass of a single particle times the number density.

The relevant factors are

The mass density of the fluid: mass per volume

The speed of the fluid, or the body; to be more exact the velocity of the body relative the fluid

The size of the body

The force felt by the body (the so called drag) should be a function of the above three factors.

Dimensional analysis

$[\rho] = ML^{-3}$ the mass density of the fluid

$[v] = LT^{-1}$ the relative velocity

$[R] = L$ a typical size of the body

$[F] = MLT^{-2}$ the force

$$[\rho^\alpha v^\beta R^\gamma F^\delta] = 1 = (ML^{-3})^\alpha (LT^{-1})^\beta L^\gamma (MLT^{-2})^\delta$$

$M: \quad \alpha + \delta = 0 \longrightarrow \alpha = -\delta$

$L: \quad -3\alpha + \beta + \gamma + \delta = 0$

$T: \quad -\beta - 2\delta = 0 \longrightarrow \beta = -2\delta$

$3\delta - 2\delta + \gamma + \delta = 0 \qquad \gamma = -2\delta$

$\left(\dfrac{F}{\rho v^2 R^2}\right)^\delta$ is the dimensionless quantity

$$Q = \frac{F}{\rho v^2 R^2}$$ The dimensionless quantity

The result of the dimensional analysis:

$$\frac{F}{\rho v^2 R^2} = c \longrightarrow$$ a constant

Dimensional analysis doesn't give the value of this constant.

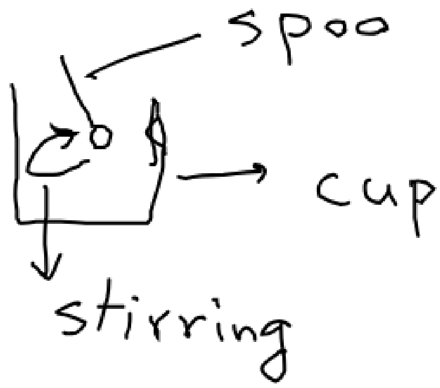
$$F = c\,\rho v^2 R^2$$

The drag (the part of the drag caused by collision)

Is there any other source (cause) for the drag?

Compare water and honey

Stirring honey with a spoon is much more difficult than strirring water with a spoon



Honey is slightly denser than water, but not very much.
So the difference in the the density doesn't explain the huge difference is the drag.

In fact, there are cases that a fluid with a lower density produces higher drag:
Oil is lessdense than water, but with same size and relative velocity produces
a bigger drag.

So there should be anaother cause for the drag.

Consider the flow of a fluid when the velocity of the fluid depends on the position:

$\longrightarrow$ bigger speed

$\rightarrow$ smaller speed

two layers

Something like friction, which tries to decrease the difference between the velocities.

$\leftarrow$ The force applied to the upper layer

$\rightarrow$ The force applied to the lower layer

This force, of course, depends on the difference of the velocities of the layers.
This difference also depends on the distance of the layers.
This distance is arbitrary. A quantity which does not depend on this arbitrariness,
Is the ratio of the velocity difference to the distance: more exactly, the limit of
this ratio when the distance tends to zero. This is the derivative of the velocity
with respect to the position:

y direction

x direction

$y+\Delta y \longrightarrow \quad v_x + \Delta v_x$

$y \longrightarrow \quad v_x$

$$\frac{\Delta v_x}{\Delta y} \xrightarrow[\Delta y \to 0]{} \frac{dv_x}{dy}$$

This should be a relevant quantity

Another relevant quantity should be the size of the layer:

S_1 S_2 surface area

$\uparrow$ $\uparrow$

F_1 F_2 the total force F

the forces $F = F_1 + F_2$

Doubling the surface area doubles the force

More generally, multiplying the surface area of the layer by alpha, results in the force multipled by the same alpha.
This means that the force is proportional to the surface area.

The force F is proportional to the surface area S

F also depends on the derivative of the velocity. This derivative is denoted by w. So F is a function of w.

But what kind of a function?

$F(w=0)=0$

Assuming that F is a smooth function of w, that is F and its derivatives are continuous,

$$F(w) = F(0) + \left\{F'(0)\right\} w + \left\{\frac{F''(0)}{2!}\right\} w^2 + \cdots$$

The Taylor series expansion of F around 0 (zero).

$F(0)$ is zero

if w is small :

$$|F'(0)| \gg |w| \; |F''(0)|$$

Then $F(w) \simeq [F'(0)]\, w$

$F(w)$ is proportional to w.

This force is proportional to the surface ~~layer~~ area, and is alos proportional to the derivative of the velocity.

The result

$$F = \eta S v' \qquad v' = \frac{dv_x}{dy}$$

F: the force

η: a parameter depending on the fluid

S: the surface area

v': the derivative of the velocity

η is called the viscosity

The relevant quantities:

The (relative) velocity v

A typical size of the body. This typical size is denoted by R.

The viscosity eta of the fluid

And, of course, the force F itself

F, R, v, η

$F = \eta S v' \quad [F] = MLT^{-2} = [\eta][S][v']$

$[S] = L^2 \quad [v'] = [v]/L = T^{-1}$

$[\eta] = M L^{-1} T^{-1}$

$$1 = \left[F^{\alpha} R^{\beta} (v)^{\gamma} \eta^{\delta}\right] = (MLT^{-2})^{\alpha} L^{\beta} (LT^{-1})^{\gamma}$$
$$\times (ML^{-1}T^{-1})^{\delta}$$

$M: \quad \alpha + \delta = 0 \quad \longrightarrow \quad \delta = -\alpha$

$L: \quad \alpha + \beta + \gamma - \delta = 0$

$T: \quad -2\alpha - \gamma - \delta = 0 \longrightarrow -2\alpha - \gamma + \alpha = 0 \nearrow \gamma = -\alpha$

$\alpha + \beta - \alpha + \alpha = 0 \quad \beta = -\alpha$

The dimensionless quantity :

$$\left(\frac{F}{\eta R v}\right)^{\alpha} \longrightarrow \frac{F}{\eta R v} = c$$

Constant

$$F = c \eta R v$$

the force of viscosity :

the force caused by viscosity

So there are two kinds of forces acting on a body which moves in a fluid:

The force due to callision.

$$F_2 = c_2 \rho L^2 v^2$$

The force due to viscosify

$$F_1 = c_1 \eta L v$$

$$\frac{F_2}{F_1} = \frac{\rho v L}{\eta} =: Re$$ Reynalds number

large Re: collision is importat

small Re: viscosity is important