

1 Heat transfer and double glazed windows

In cold days, inside a building is colder than outside. So heat goes from inside to outside. To maintain the inside temperature, heat has to be produced inside, with a rate which is equal to the rate of heat transfer to outside. This is why heaters are used in cold days.

If the walls of the building are well insulated, the rate of heat transfer to outside decreases. Hence the rate of the inside heat generation, which is needed to keep the inside temperature constant, decreases. The equation of heat transfer through conduction is similar to that of electric currents: Consider a rectangular slab of material. The heat current density (the amount of heat transfer per time per surface area) from one of the faces of the slab to the opposite face is proportional to the difference between the temperatures of those faces, and inversely proportional to the distance between them. The proportionality constant K is called the thermal conductivity of the material. Of course, heat goes from the higher temperature to the lower temperature. So,

$$J = K \frac{\Delta T}{\ell}, \quad (1)$$

where T is the temperature and ℓ is the distance between the two opposite faces. J is the heat current density, which is equal to I (the heat current) divided by A (the surface area of the each of the two faces). One then arrives at

$$I = \frac{\Delta T}{R}, \quad (2)$$

where R is the thermal resistance:

$$R = \frac{\ell}{k A}. \quad (3)$$

It is seen that the relations are similar to those of electric currents: Just substitute the thermal conductivity and the temperature difference by the electric conductivity and the voltage difference, and the heat current would be replaced by the electric current.

Similar to the case of electric circuits, when a heat current passes through successive layers of materials (resistors in series) the thermal resistances are added to produce an equivalent thermal resistance, and when the same temperature difference is applied to several layers (resistors in parallel) the conductances (the inverse resistances) are added to produce the equivalent conductance. For a building, each of the walls are subjected to essentially the same temperature difference (assuming the the outside temperature is the same everywhere, and so is the inside temperature). So the wall resistances are in parallel, meaning that the conductances are added to each other. the result is that if some of the conductances are much larger than the others (some of the resistances are much smaller than the others) then the others are essentially irrelevant.

By the way you have noticed the difference between conductivity and conductance: Conductivity is an intrinsic property of the material and is independent of the size and shape of the material. Conductance depends on both the type of the material and its size and shape.

The boundaries between inside and outside consist of solid walls and windows (or doors). The conductivities of glass and the building materials of the walls are similar to each other. After all, glass is essentially sand. But the thickness of a typical window glass is much smaller than the thickness of a typical wall. The thickness of a typical window glass is a few millimeters, while a thickness of a typical wall is tens of centimeters, about two orders of magnitude larger. This means that the conductance of a window glass is roughly two orders of magnitude larger than that of a wall with the same surface area. To be more exact, if the thickness of the glass is 4 mm and the thickness of the wall is 20 cm, then the ratio of the conductances is 50. In other words, a window of the surface area A is equivalent to a wall of the surface area $(50 A)$, regarding the thermal conductance. In most cases, 50 times the total surface area of the windows of a building is of the same order or larger than the total surface area of the walls of the same building. So windows are essential in the heat which goes outside (is wasted).

The thermal conductivity of glass is about $0.8 \text{ W m}^{-1} \text{ K}^{-1}$. So the heat current density through a simple window with a glass of the thickness of 4 mm, and with a temperature difference of 10 K, is about 2 k W m^{-2} . If the total surface area of the windows is about 10 m^2 (probably typical of a moderate flat), then the total heat current through the windows is about 20 kW.

Question: If the inside temperature is kept constant by burning some fuel (directly, or indirectly), what is rate of fuel consumption corresponding the above current, assuming that the fuel is natural gas.

Things change simple windows are replaced by double glazed windows. A double glazed window consists of two glass layers, surrounding a layer of air (or better vacuum). The thickness of the middle layer (air or vacuum) is of the same order of the thickness of the glass layers, but the thermal conductivity of air is several tens of times smaller than that of the glass. So for a double glazed window, the conductance per surface area is essentially of the same order of the same quantity for a typical wall: the differences between thickness and conductivity roughly cancel each other.

Bottom line, using double glazed windows reduces the heat wasted through the windows by factors of tens. It essentially removes that source of heat waste.