

1 Is the density of water constant?

The answer depends on the precision of the measurement. But if the density of water was not constant, what could make it vary? Temperature and pressure. It is known that materials expand if they are heated. Well, in most cases they expand. Water itself provides an exception. It contracts upon heating, if the temperature is between 0°C and 4°C. Also, squeezing a piece of matter could decrease its volume. This is certainly the case for gases. As the mass remains constant when the temperature or pressure change, expansion (an increase in the volume) means a decrease in the density; while contraction (a decrease in the volume) means an increase in the density. So the density of materials decreases (in most cases) when the temperature increases, and increases when the pressure increases. But does that happen effectively for water as well?

This is where one needs numbers. The bulk modulus of a material is minus the change of pressure divided by the the relative change of the volume of that material (when the temperature is kept constant). The (volume) thermal expansion coefficient is the relative change of the volume divided by the change of the temperature (when the pressure is kept constant). Mathematically,

$$B = -\frac{V \Delta P}{\Delta V}, \quad (1)$$

$$\lambda = \frac{\Delta V}{V \Delta T}, \quad (2)$$

where V , P , and T are the volume, pressure, and temperature, respectively; and the bulk modulus and the thermal expansion coefficient have been denoted by B and λ , respectively.

The mass is the product of the volume and the density. So if the mass is constant, that product is constant as well:

$$V \rho = \text{constant}, \quad (3)$$

where the density has been denoted by ρ . The above relation results in

$$\rho \Delta V + V \Delta \rho = 0. \quad (4)$$

So,

$$\frac{\Delta \rho}{\rho} = -\frac{\Delta V}{V}. \quad (5)$$

Using this, one arrives at

$$B = \frac{\rho \Delta P}{\Delta \rho}. \quad (6)$$

$$\lambda = -\frac{\Delta \rho}{\rho \Delta T}. \quad (7)$$

For the water, B is about 2×10^9 Pa, and λ is about 2×10^{-4} K, for not very extreme temperatures and pressures. That means if one changes the pressure by say 100 atmospheres, which is 10^7 Pa, the relative change in the volume (or the density) would be less than 1 percent. That sort of pressure happens in the oceans, at the depth of 1 km. A change of temperature by 100°C , results in 2 percent relative change in the density or the volume. So in the kitchen we cannot change the density or volume of the water significantly. That is why people say water is incompressible. In fact almost all of the liquids and solids are incompressible, in the same sense.

But isn't there any place where one can see the change in the volume of the water? What about the oceans on the earth? It is said there is a global warming. that should rise the temperature of the oceans and expand the water, which in turn, makes the surface of the oceans rise. Suppose that the temperature of the oceans increases by just 1°C . And assume that the average depth of the oceans is several kilometers. By how much does this rise the surface of the oceans?