1 How does the pressure in the atmosphere depend on the altitude

It is said that at high altitudes, it is difficult to breath, as the concentration of oxygen is less. Why is that so?

Consider an atmosphere which is in mechanical equilibrium. This means total force exerted to for each part of it is zero. The z-axis is taken to be vertical, upward. For a short cylinder between z and $(z + \Delta\, z)$, the relevant forces are gravity and the forces due to the pressure. The vertical component of the force, in the upward direction, is

$$F = A\,, P(z + \Delta\, z) - A\, P(z) - (A\, \Delta\, z)\, \rho\, g, \tag{1}$$

where A is the surface area of the cross section, P is the rpessure, ρ is the density of the atmosphere, and g is the gravitational acceleration. F should be zero. So,

$$\frac{\mathrm{d}\, P}{\mathrm{d}\, z} = -g\, \rho. \tag{2}$$

Actually this equation holds for any fluid at equilibrium, if the only forces are gravity and the pressure forces. For a liquid, the density ρ is essentially constant: fluids are incompressible. But for a gas this is not the case. Assuming that the gas in the atmosphere satisfies the equation of state for ideal gases (which it does, to a very good approximation), one has

$$P\, V = n\, R\, T, \tag{3}$$

where V is the volume of the sample, n is the number of moles in the sample, T is the (absolute) temperature, and R is the universal gas constant. Denoting the molar mass with M, one has

$$\rho = \frac{M\, n}{V}. \tag{4}$$

So,

$$\rho = \frac{M}{R\, T}\, P. \tag{5}$$

Putting this in the equation for the pressure, one arrives at

$$\frac{\mathrm{d}\, P}{\mathrm{d}\, z} = -\frac{g\, M}{R\, T}\, P. \tag{6}$$

Everything on the right-hand side is constant, apart from P (which is the unknown to be found) and T. T does depend on the position, altitude, and time. But up to altitudes of a few kilometers, the temperature usually doesn't go below say (−50°C). Also, there are not many places on the earth where the temperature exceeds say (+50°C). So it is safe to assume that the absolute temperature in between say (220 K) and (320 K). This means the temperature could vary up to a few ten percents of its average value. To proceed, it is taken constant. Using these, one arrives at

$$\frac{\mathrm{d}\,P}{\mathrm{d}\,z} = -\frac{1}{L}\,P, \tag{7}$$

where

$$L = \frac{R\,T}{g\,M}. \tag{8}$$

The solution to the differential equation for the pressure is then

$$P(z) = P(0)\,\exp\left(-\frac{z}{L}\right). \tag{9}$$

This means that the atmospheric pressure at an altitude L, is $(1/e)$ times the atmospheric pressure at the sea level. The average value of M can be obtained as follows. the atmosphere consists of 78% nitrogen with the molar mass ($28\,\mathrm{g\,mol^{-1}}$), and 21% oxygen with the molar mass ($32\,\mathrm{g\,mol^{-1}}$), and 1% other gases, mostly argon. The average molar mass is then ($29\,\mathrm{g\,mol^{-1}}$). Taking the temperature to be about (0°C), or (270 K), one arrives at.

$$L = 8\,\mathrm{k\,m}. \tag{10}$$

The altitude of Tehran varies between (1100 m) and (1800 m). (It is illegal to build at altitudes higher than (1800 m).) So if the atmospheric pressure at the sea level is (760 m m Hg), the atmospheric pressure in Tehran should be between these two values:

$$P(z = 1.1\,\mathrm{k\,m}) = 660\,\mathrm{m\,m\,Hg}. \tag{11}$$

$$P(z = 1.8\,\mathrm{k\,m}) = 610\,\mathrm{m\,m\,Hg}. \tag{12}$$

Of course the pressure changes with time, and these are just approximate values. But it is true that atmospheric pressure at the highest point of Tehran is about 0.8 times the atmospheric pressure at the sea level. The atmospheric pressure at the peak of mount Everest, an altitude of about (9 k m), should be about 0.3 times the atmospheric pressure at the sea level.