1 Examples of scaling, II

Warm blooded animals are those which keep their body temperature essentially constant. Mammals and birds are warm blooded. Cold blooded animals are those for which the body temperature varies significantly.

A warm blooded animal keeps it body temperature constant by consuming energy. Most of the energy such an animal consumed is used for that, and goes away to the environment. That is why the energy (hence the food) a mammal needs is much more than that of a reptile of same size.

Let's concentrate on warm blooded animals, to be more specific on mammals. What is the ratio of the power consumed by a tiger, to that of a cat? What is the relation of this ratio that the ratio of their masses? Assuming that the whole power consumed by the animal goes away as heat to the environment, as the dissipated heat is proportional to the surface area of the animal's skin, the consumed power should be proportional to the surface area of the skin, hence proportional to the square of the length size:

$$P \propto L^2. \tag{1}$$

P and L are the consumed power and the length size, respectively. V (the volume of the animal) is proportional to L^2, and M (the animal's mass) is proportional to V, assuming the densities of mammals are essentially the same, in fact essentially the same as the density of water. Hence,

$$P \propto M^{2/3}. \tag{2}$$

In reality, the above assumptions are, as expected, too simplified. However, experimental results show that the scaling relation remains intact, but with an exponent *alpha* which is different from (2/3):

$$P \propto M^{\alpha}, \tag{3}$$

where α is closer to (3/4) than to (2/3):

$$\alpha \approx 0.73. \tag{4}$$

The quantity p is defined as the consumed power per mass:

$$p = \frac{P}{M}. \tag{5}$$

So,

$$p \propto M^{-(1-\alpha)}. \tag{6}$$

As α is less than 1, the exponent in the above relation is negative: the consumed power per mass decreases as the mass increases.

But what is the significance of this quantity p? The cells of bigger animals are similar to the cells of smaller ones (except for some very specific cells like some neurons which may be longer in bigger animals). So regarding the cells, the difference between small and large animals are not in their cell sizes but in the numbers of their cells. The fact that the consumed power per mass is smaller in bigger animals, means that the cells of bigger animals consume energy more slowly that the cells of smaller animals. Aging is a result of accumulation of errors in the chemical reactions occurring in the cells. One could say that the rate of aging is proportional to the rates of those reactions. The rates of those reactions themselves are proportional to the consumed power (in the cell). So the rate of aging is proportional to the consumed power per cell, or consumed power per mass. The life span of an animal is inversely proportional to the rate of aging. Putting all these together, one arrives at a simple relation for T (the life span of a mammal) in terms of M (its mass):

$$T \propto M^{1-\alpha} \tag{7}$$

- Check this relation for human, elephant, and blue whale. Find the data (average mass and average life span) you need.
- Does human seem to be an anomaly?