1 Examples of scaling

There are many cases where the dependence of a quantity Y on a quantity X is scaling.

Consider D, the thickness of the leg of animal, compared to H, its height. For animal which are approximately similar, the volume V is proportional to H^3. Hence the weight W is proportional to H^3:

$$W = c\,\rho\,g\,H^3, \tag{1}$$

where ρ and g are the body density and the gravitational acceleration, respectively, and c is a dimensionless constant. The pressure P at the foot of the animal is equal to its weight divided by the surface area of the foot. The latter is proportional to D^2. So

$$P = c'\rho\,g\,H^3\,D^{-2}. \tag{2}$$

As the material from which the body of similar animals (a horse and an elephant, for example) is built is essentially the same, the pressure that similar animals can tolerate is essentially the same. Keeping the pressure (and the density) constant, one arrives at

$$D = \alpha\,g^{1/2}\,H^{3/2}, \tag{3}$$

where α is a (dimensionfull) constant. Of course on the earth, g is also constant. So D is proportional to $H^{3/2}$. That means for larger animals, the thickness of the legs relative to the height is larger:

$$\frac{D}{H} = \alpha\,g^{1/2}\,H^{1/2}. \tag{4}$$

have you notices that an elephant has a thicker leg (compared to its height) than a horse?

Now, the elephant is the biggest land animal. Could we have yet bigger land animals? Is there any restriction for how big land animals could get? The relation between the leg width and the height suggests such a restriction. Consider the relation of D with L, the lateral size of the animal. Using similar arguments, one would arrive at a similar relation between D and L:

$$\frac{D}{L} = \alpha'\,g^{1/2}\,L^{1/2}. \tag{5}$$

But D cannot exceed L. Consider two animals: a real one (0) and a hypothetical one (1), where for the latter D is equal to L:

$$\frac{D_1}{L_1} = 1. \tag{6}$$

using the relation of D with L, one arrives at

$$\frac{(D_1/L_1)}{(D_0/L_0)} = \left(\frac{L_1}{L_0}\right)^{1/2}. \tag{7}$$

So,

$$L_1 = \frac{L_0}{(D_0/L_0)^2}. \tag{8}$$

If I estimate the ratio (D_0/L_0) for the elephant about 0.5, Then

$$L_1 = 4\,L_0. \tag{9}$$

the maximum possible (length) size would be some 4 times that of the elephant.

This restriction is much different for animals which live in water. Have you noticed that the largest animal (in fact the largest animal the earth has ever seen) is living in the ocean? We are talking about the blue whale.

The reason For keeping g explicit (not hiding it in a dimensionfull constant) is that this allows for some speculation. What would happen if animals on other planets were investigated? Please forget about other possible differences and the fact that up to now no other planet apart from the earth has been discovered with life on it. Just consider a different g. You see that the maximum size is inversely proportional to g. So if there was life on the moon (with oxygen and other things required for life taken acre of of), larger animals (compared to the earth) would have been possible. The acceleration of gravity on the surface of the moon is about one sixth that of the earth. That would allow for animals six time bigger. Imagine a creature (6×4) the height of an elephant walking on the moon.

A gauge for human obesity is a quantity called BMI (body mass index). This quantity is defined as the mass in terms of kilogram divided by the square of the height in terms of meter:

$$\mathrm{BMI} = \frac{M/(\mathrm{k\,g})}{(H/\mathrm{m})^2}, \tag{10}$$

where M is the mass. Why is this quantity important? We have already seen that the pressure experienced by the feet is proportional to the weight divided by the square of the leg width. This is also true for the pressure at say half (or any other fraction) of the height. For humans, one can assume that the leg width is proportional to the height. So instead of the square of the leg width, one can use the square of the height. Then it is seen that the pressure at any given fraction of the height is proportional to the BMI.

- What is your BMI?
- Find the different ranges of BMI, think, normal, fat, etc.