1 The time needed to cook something in boiling water

When something, say an egg, is inside boiling water, the temperature of the surface of that thing is that of the boiling water, which is essentially fixed. But the temperature inside that thing rises gradually. For that thing to be cooked, the temperature everywhere inside the thing should pass a ceratin minimum, and that takes time.

The bigger the thing, the larger the time needed that the inside temperature reach that minimum. But how large? That is, what is the relation of the required time to the size of the thing? This is an example of scaling, as it will be seen.

In order that something get hotter, it should be heated, that is, it should receive energy. Heat flows from hot points to cold points. The heat current through a surface (the amount of energy passed through that surface divided by time) is proportional to the surface area and derivative of the temperature with respect to the position. The proportionality constant is called the thermal conductivity, K. The dimension of the heat current is the dimension of energy divided by the dimension of time. The dimension of the surface area is the square of the dimension of length. The dimension of the derivative of the temperature with respect to the position is the dimension of temperature divided by the dimension of length. Putting all these together, it is seen that

$$(M\,L^2\,T^{-2}/T) = [K]\,L^2\,(\Theta\,L^{-1}), \tag{1}$$

where M, L, T, and Θ are the dimensions of mass, length, time, and temperature, respectively. Simplifying the above,

$$[K] = M\,L\,T^{-3}\,\Theta^{-1}. \tag{2}$$

When something receives heat, its temperature rises, The heat is proportional to the temperature change, and the proportionality constant is called the heat capacity, C. For a larger body, a larger amount of heat is needed to produce the same temperature difference. In fact the heat needed to produce a fixed temperature difference is proportional to the volume (or the mass) of the body. This means C is proportional to the volume (or mass). The heat capacity divided by volume, is then independent of the size of the body. It does depend on the type of the matter. the heat capacity divided by the volume is denoted by c, and is called the volume specific heat capacity. It is seen that

$$M\,L^2\,T^{-2} = [C]\,\Theta. \tag{3}$$

$$[C] = [c]\,L^3. \tag{4}$$

So,

$$[c] = M\,L^{-1}\,T^{-2}\,\Theta^{-1}. \tag{5}$$

Now we are ready to perform a dimensional analysis ro relate the time needed to cook something to the size of that thing. the parameters involved are the time t and the size ℓ, and the two dimensionful constants K and C. It is seen that there is only independent dimensionless parameter which can be constructed with these:

$$Q = t\,\ell^{-2}\,K\,c^{-1}. \tag{6}$$

So the result of the dimensional analysis is that

$$Q = \alpha, \tag{7}$$

where α is a dimensionless constant. The relation of t with ℓ then becomes

$$t = \alpha\,K^{-1}\,c\,\ell^2, \tag{8}$$

or,

$$t \propto \ell^2. \tag{9}$$

As an example, let's estimate the time needed to cook an ostrich's egg. The material of an ostrich's egg is essentially the same as that of an ordinary egg. So their heat conductivities are the same, and their specific heat capacities are also the same. The mass of an ostrich's egg is about $(1\,\mathrm{k\,g})$, which is about 30 times the mass of an ordinary egg. So the length size of an ostrich's egg is about $30^{1/3}$ or 3 times the length size of an egg. The square of the ratio of the length sizes is then about 10. This means that the time needed to cook an ostrich's egg is about 10 times the time needed to cook an ordinary egg. The latter is about 10 minutes. So the time needed to cook an ostrich's egg is about one and a half hours.