1 Why are planets spherical, or what is the maximum height of a mountain on the earth?

Of course the meaning of "why are planets spherical", is not the same as "what is the maximum height of a mountain on the earth". But these questions are related to each other. The point which is common in both phenomena, is that when the pressure inside a solid exceeds the so called yield point, the solid softens and undergoes a permanent (irreversible) change in its shape. So a piece of rock may behave like a paste.

A planet is a large piece of solid. It could also have liquid parts, like the outer nucleus of the earth. When the planet is formed, as a result of random collisions between small chunks of matter, its shape should be irregular. That is the case, until the time the planet becomes so big that the pressure inside it exceeds the yield point. At that time, parts of the planet become soft and the shape of the planet no longer remains fixed. The result is a spherical planet.

But when does the pressure exceed the yield point? A crude estimation is to take both the density ρ of the material and the gravitational acceleration g due to the planet constant. Then at the depth h, the pressure P would be

$$P = \rho\, g\, h. \tag{1}$$

So the maximum pressure would be at the center with $h = R$:

$$P_{\max} \sim \rho\, g\, R. \tag{2}$$

The gravitational acceleration at the surface of the planet is

$$g \sim \frac{G\, M}{R^2}, \tag{3}$$

where G is universal constant of gravitation, and M is the mass of planet. It seems I have already used the spherical shpae of the planet and its radius R. But for a crude estimation, one could use the *size* of the planet instead of its radius. Denoting the size by the same R, it is related to the mass and the density through

$$M \sim \rho\, R^3. \tag{4}$$

So,

$$P_{\max} \sim G\, M^{2/3}\, \rho^{4/3}. \tag{5}$$

For fixed density, the pressure increases as M increases. There is a value M_c for M, at which the maximum pressure becomes equal to the yield point P_y. When the mass of the body exceeds M_c, softening occurs and the body becomes spherical. It is seen that

$$M_c \sim G^{-3/2}\, P_y^{3/2}\, \rho^{-2}. \tag{6}$$

Alternatively, one could express this in terms of size $R_{\rm c}$, that if it is exceeded by R, the softening occurs:

$$R_{\rm c} \sim G^{-1/2}\, P_{\rm y}^{1/2}\, \rho^{-1}. \tag{7}$$

These two estimations could also be obtained using dimensional analysis: What is the relation between G, $P_{\rm y}$, ρ, and $M_{\rm c}$ or $R_{\rm c}$?

Regarding $P_{\rm y}$, one could find its values for different materials from handbooks or other similar resources, or again estimate an order of magnitude for it. To do the latter, one needs an order of magnitude for the binding energy between the atoms (or ions) constructing the matter, and an order of magnitude for the distance between the neighboring atoms (or ions). The only quantity with the dimension of pressure which can be constructed from the binding energy and the distance, is the binding energy divided by the distance to the power three. The former is of the order of some tenth of ($e\,{\rm V}$). The distance between neighboring atoms or (ions) in a solid is of the order of a few Å. So, using

$$e\,{\rm V} = 1.6 \times 10^{-19}\,{\rm J}, \tag{8}$$

$$\text{Å} = 10^{-10}\,{\rm m}, \tag{9}$$

one arrives at

$$P_{\rm y} \sim 10^{10}\,{\rm Pa}. \tag{10}$$

One has

$$G = 6.7 \times 10^{-11}\,{\rm m}^3\,{\rm s}^{-2}\,({\rm k\,g})^{-1}, \tag{11}$$

For rockey planet, like the earth, the density is a few times the density of water. The density of water is ($10^{-3}\,{\rm k\,g\,m^{-3}}$). Let's use the following estimations.

$$G \sim 10^{-10}\,{\rm m}^3\,{\rm s}^{-2}\,({\rm k\,g})^{-1}. \tag{12}$$

$$\rho \sim 10^{4}\,{\rm k\,g\,m^{-3}}. \tag{13}$$

The final results are

$$M_{\rm c} \sim 10^{22}\,{\rm k\,g}. \tag{14}$$

$$R_{\rm c} \sim 10^{6}\,{\rm m}. \tag{15}$$

Of course these are not exact results. For comparison, $M_{\rm e}$ and $R_{\rm e}$, the mass and the radius of the earth, respectively, are

$$M_{\rm e} = 6 \times 10^{24}\,{\rm k\,g}. \tag{16}$$

$$R_{\rm e} \sim 6 \times 10^{6}\,{\rm m}. \tag{17}$$

What about the other question, the maximum possible height of a mountain on the earth? Well, again the pressure at the bottom of the mountain should not exceed the yield point. Using (1), this time with h being the height of the mountain, P shouldn't exceed P_{y}. So the maximum value for h is h_{c}, with

$$h_{\mathrm{c}} = \frac{P_{\mathrm{y}}}{\rho\, g}. \tag{18}$$

Using

$$g = 10\,\mathrm{m\,s^{-2}}, \tag{19}$$

and (10) and (13), one arrives at

$$h_{\mathrm{c}} \sim 10^5\ \mathrm{m}. \tag{20}$$

Again this is not exact. For comparison, h_{E}, the height of mount Everest, is about (9000 m):

$$h_{\mathrm{E}} \sim 10^4\ \mathrm{m}. \tag{21}$$