1 How to measure the number of molecules, II

Another question, seemingly unrelated to the measurement of the number of molecules, is this. Why is the sky blue? Before that, why isn't the sky black? That the sky is not black, means that from a direction different from the direction of the sun, some light comes towards us. But why? If the source of the light is the sun, then light should come only from the direction of the sun. And that is, in fact, the case where there is no atmosphere. For example, on the moon the sky is black. No light comes except from the direction of the sun. So the key point on the earth is the existence of the atmosphere. The light from the sun reaches the molecules of the atmosphere, and some of it is scattered. That is, its direction is changed. It is this scattered light which reaches us from direction other than the direction of the sun, so that those directions don't look black. But then, why don't those directions look white (or yellow) like the color of the sun, and instead look blue? It is known that the light from the sun consists of lights of different colors. If the relative strengths of different colors is changed, the overall color will change. For example, if the violet-blue parts are enhanced relative to red-orange parts, then the color is shifted towards blue.

The scattering of light happens like this. Light is an electromagnetic wave: it consists of oscillating electric and magnetic fields. These fields exert an electromagnetic force on the charged particles within the molecules, making them oscillate as well. An accelerated charge radiates an electromagnetic wave, hence behaves like a light source. So the atmospheric molecules illuminated by the sun, behave like tiny light sources. They get some of the energy of incoming light, and emit it in different directions. This is light scattering, which obviously doesn't happen unless these molecules are actually there. So this doesn't happen on the moon, as the moon has a negligible atmosphere, essentially no molecules to scatter the light. This explains why the sky is black on the moon but not so on the earth. But still, one has to explain why it is blue on the earth. To do so, one should find a relation between the strength of the scattering, and the wavelength (or frequency) of the light.

It was pointed out (without proof, but let's accept it) that an accelerated charged particle radiates energy. One can estimate the radiated power through a dimensional analysis. The power $\mathcal{P}$ is to be related to the acceleration a, and the charge q. there are two dimensional constants which are related to the problem: c (the speed of light, after all we are studying the light) and the constant of the electric force. This latter (K) is related to ε_0 (the permitivity of the vacuum):

$$K = \frac{1}{4\,\pi\,\varepsilon_0}. \tag{1}$$

The result of the dimensional analysis is that

$$\mathcal{P} \propto K\,c^{-3}\,q^2\,a^2. \tag{2}$$

The acceleration of a harmonic oscillator (like that of a mass attached to a spring) is the displacement times the square of the angular frequency. The charge times the displacement is the electric dipole corresponding to the charge. So the above relation becomes

$$\mathcal{P} \propto K\, c^{-3}\, p^2\, \omega^4, \tag{3}$$

where ω is the angular frequency. This shows that the scattered power is proportional to the fourth power of the angulare frequency. using the relation of the wavelength λ with the angular momentum:

$$\lambda\, \omega = 2\, \pi\, c, \tag{4}$$

one can express the scattered power as

$$\mathcal{P} \propto K\, c p^2\, \lambda^{-4}. \tag{5}$$

So, why is the sky bluer than the sun? Because the color of the sky is the color of the scattered sun light, and smaller wavelengths (the violet-blue part) are scattered more strongly than the larger wavelengths (the red-orange part).

This explanation for the color of the sky, explains another phenomenon as well. At sunrise and sunset, the sun itself looks red and not yellow (or white). the reason is again the scattering of the sun light by the atmosphere. Recall that the atmosphere molecules radiate some energy. But they don't produce the scattered energy themselves. They get this energy from the initial source, which the sun. So as the sun light moves through the atmosphere, its intensity along its direct path is reduced, as some of its energy is taken by the molecules to be scattered in other direction. So, when we look at the sun, we see only a decreased intensity. It was already seen that scattering is stronger at smaller wavelengths. So this decrease in the intensity of the direct sun light is more important in the violet-blue part, that the red-orange part. The sun looks redder that it actually is.

But why this happens only at sunrise or sunset? shouldn't one always see the sun redder than it actually is, so that there is no difference between the color of the sun at sunrise or sunset, and the color of the sun at say noon? The point is that the strength of the scattering depends also on the number of the scatterers (the atmospheric molecules which scatter the light). The atmosphere is very thin, compared to the size of the earth. When the sun is considerably above the horizon (it is not sunrise or sunset), ℓ (the length of the path of the sun light in the atmosphere) is obtained through

$$\ell = \frac{h}{\cos\theta}, \tag{6}$$

whree h is the effective thickness of the atmosphere (about $10\,\mathrm{k\,m}$), and θ is the angle between the direction of then sun and the radial direction away from the earth's center. This relation is obviously wrong for the sunrise and sunset, when θ is $(\pi/2)$. the correct relation at those times is

$$\ell = \sqrt{(R+h)^2 - R^2}, \tag{7}$$

where R is the radius of the earth. As R is much larger than h, the above could be approximated to

$$\ell \approx \sqrt{2\,R\,h}. \tag{8}$$

So the ratio of the atmospheric path length near the sunrise or the sunset, to the atmospheric path length at other times, is of the order of $\sqrt{R/h}$, about 30. A larger atmospheric path length means a larger number of scatterers, hence a stronger scattering. That means that effect of scattering is bigger near sunrise and sunset, compared to other times. the difference is so that, at sunrise or sunset it is easy to look at the sun directly, as the direct sun light is weak. It is not so easy at other times. That also means that the color-changing effect of the scattering is stronger at sunrise or sunset. Hence the reddening of the sun is more pronounced at sunrise and sunset. the sun looks redder at sunrise and sunset, relative to other times.